Combining equation (27) with the solid mechanics of (23) and the geometry of (4), one arrives at the desired formula for the slot depth h:

$$h = 2\,\mu_w \frac{d_o P_o}{\tau_o} \int_0^{\theta_o} \frac{e^{\mu_w(\theta - \theta_o)} \sin\theta}{1 + (v/c)\sin\theta}\, d\theta \quad , \tag{28}$$

provided that P_o is greater than the critical pressure P_c of (25). Actually P_o must be much larger than P_c for (28) to be strictly valid. According to (28) and (25), the ratio P_o/P_c is of order h/d_o, and d_o/h has been assumed small throughout the analysis. That is why P_o rather than $(P_o - P_c)$ appears in (28), which otherwise is compatible with the empirical formula (1) implicit in the work of Zelenin, Vesselov, and Koniashin.

Apparently (28) cannot be evaluated in terms of elementary functions, but elementary forms are available for the important limits $v/c \to 0$ and $v/c \to \infty$. Thus as $v/c \to 0$,

$$h \to \frac{2\mu_w}{1 + \mu_w^2} \frac{d_o P_o}{\tau_o} \left(\mu_w \sin\theta_o - \cos\theta_o + e^{-\mu_w \theta_o}\right) \quad , \tag{29}$$

and in the opposite limit $v/c \to \infty$,

$$h \to \frac{2k d_o P_o}{\mu_r g v} \left(1 - e^{-\mu_w \theta_o}\right) \quad . \tag{30}$$

The intrinsic speed c has been eliminated from (30) by means of definition (22) to show that slot depth ceases to depend on shear strength at high feed rates. Notice that neither (29) nor (30) reaches a maximum under normal impingement, $\theta_o = \pi/2$ radians. The optimum angle of impingement depends on μ_w and v/c but always lies between $\pi/2$ and π.